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Elementary Statistics
A Step by Step Approach Seventh Edition

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| CHAPTER 6 |
| :---: |
| The Normal Distribution |



## Objectives

$\square$ Identifying distributions as symmetrical or skewed. $\square$ Identifying the properties of the normal distribution.
$\square$ Finding the area under the standard normal distribution, given various $Z$ values.
$\square$ Finding the probability of a normally distributed variable by transforming it into a standard normal variable.
$\square$ Finding specific data values for given percentages using the standard normal distribution.
Using the central limit theorem to solve problems involving sample means for large samples.

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Notes


| Introduction |
| :--- |
| $\square$ Many continuous variables have distributions that are |
| bell-shaped and are called approximately normally |
| distributed variables, such as the heights of adult men, |
| cholesterol level of adults, etc... |
| $\square$ A normal distribution is also known as the bell curve |
| or the Gaussian distribution. |



## Normal and Skewed Distributions

- A normal distribution is a continuous, symmetric, bell-shaped distribution of a variable.
$\square$ If the data values are evenly distributed about the mean, the distribution is said to be symmetrical. (mean $=$ median $=$ mode )
$\square$ If the majority of the data values fall to the left or right of the mean, the distribution is said to be skewed.
- See Figures 6-1 and 6-2 page 301.

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## Normal Distribution Properties

$\square$ The shape and position of the normal distribution curve depend on two parameters, the mean and the standard deviation.


Notes


## Normal Distribution Properties

$\square$ The mean, median, and mode of the normal distribution are equal and located at the center of the distribution.

- The normal distribution curve is unimodal (i.e., it has only one mode).
$\square$ The curve of the normal distribution is continuous, i.e., there are no gaps. Thus, for each value of $X$, there is a corresponding value of $Y$.

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## Normal Distribution Properties

$\square$ The total area under the normal distribution curve is equal to 1.00 or $100 \%$.

- The area under the normal curve that lies within $\checkmark$ one standard deviation of the mean is approximately 0.68 (68\%).
$\checkmark$ two standard deviations of the mean is approximately 0.95 (95\%).
$\checkmark$ three standard deviations of the mean is approximately 0.997 (99.7\%).

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Normal Distribution Properties
Notes

- Areas Under the Normal Curve




## Standard Normal Distribution

$\square$ The standard normal distribution is a normal distribution with a mean of 0 and a standard deviation of 1 .

- See examples 6-1 - 6-5 pages 306-309.
- All normally distributed variables can be transformed into the standard normally distributed variable by using the $z$ value which is the number of standard deviations that a particular $x$ value is away from the mean

$$
z=\frac{\text { value }- \text { mean }}{\text { standard deviation }} \text { or } z=\frac{x-\mu}{\sigma}
$$

$\square$ See examples 6-6-6-8 pages $317-319$.


## Area Under the Standard Normal distribution Curve

$\square$ The table of the standard normal distribution gives the probability to the left of the values, thus $P(z<a)$.

- Example: $P(z<2.32)=0.9898$



## Area Under the Standard Normal distribution Curve



## Notes



- If the income of 10000 family follows a normal distribution with mean 1800 SAR and standard deviation 300 SAR, find
- The probability of a family income is less than 2550.
$P(X<2550)=P\left(\frac{X-\mu}{\sigma}<\frac{2550-1800}{300}\right)=P(z<2.5)=0.9938$
- The probability of a family income is less than 1300 .
$P(X<1200)=P\left(\frac{X-\mu}{\sigma}<\frac{1300-1800}{300}\right)=P(z<-1.67)=0.0475$
- The probability of a family income is greater than 2400 .
$P(X>2400)=P\left(\frac{X-\mu}{\sigma}>\frac{2400-1800}{300}\right)=P(z>2)$

$$
=1-P(z<2)=1-0.9772=0.0228
$$

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## Notes



- If the income of 10000 family follows a normal distribution with mean 1800 SAR and standard deviation 300 SAR, find
- The probability of a family income is greater than 1500 .

$$
\begin{aligned}
P(X>1500) & =P\left(\frac{X-\mu}{\sigma}>\frac{1500-1800}{300}\right)=P(z>-1) \\
& =1-P(z<-1)=1-0.1587=0.8413
\end{aligned}
$$

- The probability of a family income is between 1650 and 2250 ,

$$
\begin{aligned}
P(1650<X<2250) & =P\left(\frac{1650-1800}{300}<\frac{X-\mu}{\sigma}<\frac{2250-1800}{300}\right) \\
& =P(-0.5<z<1.5)=P(z<1.5)-P(z<-0.5) \\
& =0.9332-0.3085=0.6247
\end{aligned}
$$

- The number of families that have income greater than 1500 ,
$P(X>1500) \times 1000=0.8413 \times 10000=8413$ family
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## Notes



- The lifetime of a one type of microwaves follows a normal distribution with mean 3 years and standard deviation 1 year. If one microwave was chosen randomly,
- What is the probability that its lifetime will be greater than 2 years?

$$
\begin{aligned}
P(X>2) & =P\left(\frac{X-\mu}{\sigma}>\frac{2-3}{1}\right)=P(z>-1) \\
& =1-P(z<-1)=1-0.1587=0.8413
\end{aligned}
$$

- If the microwaves have warranty for one year, what is the percentage of microwaves that the factory has to exchange with new ones.

$$
P(X<1)=P\left(\frac{X-\mu}{\sigma}<\frac{1-3}{1}\right)=P(z<-2)=0.0228=2.28 \%
$$

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## Notes



| Calculating the Value of $X$ |
| :---: |
| - When one must find the value of $X$, the following formula can be used:$X=z \cdot \sigma+\mu$ |
|  |  |



- To qualify for a police academy, candidates must score in the top $10 \%$ on a general abilities test. The test has a mean of 200 and a standard deviation of 20. Find the lowest possible score to qualify. Assume the test scores are normally distributed.
- The test value $X$ that cuts off the upper $10 \%$ of the area under a normal distribution curve is desired.

- To qualify for a police academy, candidates must score in the top $10 \%$ on a general abilities test. The test has a mean of 200 and a standard deviation of 20 . Find the lowest possible score to qualify. Assume the test scores are normally distributed.
- Now, to find the $z$ value that corresponds to an area of 0.9000 look up the table. If the specific value cannot be found, use the closest value - in this case 0.8997


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## Notes



- To qualify for a police academy, candidates must score in the top $10 \%$ on a general abilities test. The test has a mean of 200 and a standard deviation of 20 . Find the lowest possible score to qualify. Assume the test scores are normally distributed.
- Now, substitute in the formula $X=z \cdot \sigma+\mu$, thus

$$
x=1.28 \times 20+200=225.60 \approx 226
$$

- Hence, a score of 226 should be used as a cutoff. Anybody scoring 226 or higher qualifies.

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- For a medical study, a researcher wishes to select people in the middle $60 \%$ of the population based on blood pressure. If the mean systolic blood pressure is 120 and the standard deviation is 8 , find the upper and lower readings that would qualify people to participate in the study.
- Note that two values are needed, one above the mean and one below the mean.

- For a medical study, a researcher wishes to select people in the middle $60 \%$ of the population based on blood pressure. If the mean systolic blood pressure is 120 and the standard deviation is 8 , find the upper and lower readings that would qualify people to participate in the study.
- Now, the closest $z$ value for an area of 0.8000 (0.7995) is 0.84. Thus,

$$
\begin{aligned}
& X_{1}=-0.84 \times 8+120=113.28 \\
& X_{2}=0.84 \times 8+120=126.72
\end{aligned}
$$

- Therefore, the middle $60 \%$ will have blood pressure reading between 113.28 and 126.72.


## Distribution of Sample Means

$\square$ A sampling distribution of sample means is a distribution obtained by using the means computed from random samples of a specific size taken from a population.
$\square$ Sampling error is the difference between the sample measure and the corresponding population measure due to the fact that the sample is not a perfect representation of the population.

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## The Central Limit Theorem

$\square$ As the sample size $n$ increases, the shape of the distribution of the sample means taken with replacement from a population with mean $\mu$ and standard deviation $\sigma$ will approach a normal distribution.
$\square$ Thus, the mean of the sample means equals the population mean, $\mu_{x}=\mu$, and the standard deviation of the sample means which is called the standard error of the mean is $\sigma_{\bar{x}}=\frac{\sigma}{\sqrt{n}}$

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## The Central Limit Theorem

$\square$ The central limit theorem can be used to answer questions about sample means in the same manner that the normal distribution can be used to answer questions about individual values.
$\square$ A new formula must be used for the $z$ values:

$$
z=\frac{\bar{x}-\mu}{\sigma / \sqrt{n}}
$$

Notes


- A.C. Neilsen reported that children between the ages of 2 and 5 watch an average of 25 hours of TV per week. Assume the variable is normally distributed and the standard deviation is 3 hours. If 32 children between the ages of 2 and 5 are randomly selected, find the probability that the mean of the number of hours they watch TV is greater than $\overline{26.3}$ hours.

$$
\begin{aligned}
P(\bar{x}>26.3) & =P\left(\frac{\bar{x}-\mu}{\sigma / \sqrt{n}}>\frac{26.3-25}{3 / \sqrt{32}}\right)=P(z>2.45) \\
& =1-P(z<2.45)=1-0.9929=0.0071
\end{aligned}
$$



- The average age of a vehicle registered in the United States is 96 months. Assume the standard deviation is 16 months. If a random sample of 36 cars is selected, find the probability that the mean of their age is between 90 and 100 months.
- The $z$ values are

$$
\begin{aligned}
P(90<\bar{x}<100) & =P\left(\frac{90-96}{16 / \sqrt{36}}<\frac{\bar{x}-\mu}{\sigma / \sqrt{n}}<\frac{100-96}{16 / \sqrt{36}}\right) \\
& =P(-2.25<z<1.5)=P(z<1.5)-P(z<-2.25) \\
& =0.9332-0.0122=0.921
\end{aligned}
$$

## Notes



- The average number of pounds of meat that a person consumes a year is 218.4 pounds. Assume that the standard deviation is 25 pounds and the distribution is approximately normal.
- Find the probability that a person selected at random consumes less than 224 pounds per year.

$$
P(x<224)=P\left(\frac{x-\mu}{\sigma}<\frac{224-218.4}{25}\right)=P(z<0.22)=0.5871
$$

- If a sample of 40 individual is selected, find the probability that the mean of the sample will be less than 224 pounds per year.

$$
P(\bar{x}<224)=P\left(\frac{\bar{x}-\mu}{\sigma / \sqrt{n}}<\frac{224-218.4}{25 / \sqrt{40}}\right)=P(z<1.42)=0.9222
$$

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## Notes



## Summary

$\square$ The normal distribution can be used to describe a variety of variables, such as heights, weights, and temperatures.

- The normal distribution is bell-shaped, unimodal, symmetric, and continuous; its mean, median, and mode are equal.
- The normal distribution can be used to approximate other distributions.
- Mathematicians use the standard normal distribution which has a mean of 0 and a standard deviation of 1 .



## Summary

- The normal distribution can be used to describe a sampling distribution of sample means. These samples must be of the same size and randomly selected with replacement from the population.
- The central limit theorem states that as the size of the samples increases, the distribution of sample means will be approximately normal.
- The distribution of sample means is much less variable than the distribution of individual data value.


|  |  | 0.00 | 0.01 | 0.02 | 0.03 | 0.04 | 0.05 | 0.06 | 0.07 | 0.08 | 0.09 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| The Table of the Cumulative Standard Normal Distribution | . 3.5 | 0.0002 | 0.0002 | 0.0002 | 0.0002 | 0.0002 | 0.0002 | 0.0002 | 0.0002 | 0.0002 | 0.0002 |
|  | -3.4 | 0.0003 | 0.0003 | 0.0003 | 0.0003 | 0.0003 | 0.0003 | 0.0003 | 0.0003 | 0.0003 |  |
|  | -3.3 | 0.0005 | 0.0005 | 0.0005 | 0.0004 | 0.0004 | 0.0004 | 0.0004 | 0.0004 | 0.0004 |  |
|  | -3.2 | -.0007 | 0.0007 | 0.0006 | 0.0008 | 0.0006 | 0.0006 | 0.0006 | 0.0005 | 0.0005 | 0.0005 |
|  | -3, ${ }^{\text {a }}$ | -.0010 | 0.0009 | 0.0009 | 0.0009 | 0.0008 | 0.0008 | 0.0008 | 0.0008 | 0.0007 | 0.0007 |
|  | .3.0 | 0.0013 | 0.00013 | 0.0013 | 0.0012 | 0.0012 | 0.0011 | 0.0011 | 0.0011 | 0.0010 | 0.0010 |
|  | -2.9 | 0.0019 | 0.0018 | 0.0018 | 0.0017 | ${ }^{0.0016}$ | ${ }^{0.0016}$ | 0.0015 | 0.0015 | 0.0014 | 0.0014 |
|  | -2.8 | 0.0026 | 0.0025 | 0.0024 | 0.0023 | 0.0023 | 0.0022 | 0.0021 | 0.0021 | 0.0020 | 0.0019 |
|  | -2.7 | 0.0035 | 0.0034 | 0.0033 | 0.0032 | 0.0031 | 0.0030 | 0.0029 | 0.0028 | 0.0027 | 0.0026 |
|  | -2.6 | 0.0047 | 0.0045 | 0.0044 | 0.0043 | 0.0041 | 0.0040 | 0.0339 | 0.0038 | 0.0037 | 0.0036 |
|  | -2.5 | 0.0062 | 0.0060 | 0.0059 | 0.0057 | 0.0055 | 0.0054 | 0.0052 | 0.0051 | 0.0049 | 0.0048 |
|  | -2.4 | 0.0082 | 0.0080 | 0.0078 | 0.0075 | 0.0073 | 0.0071 | 0.0069 | 0.0068 | 0.0068 | 0.0004 |
|  | -2.3 | 0.0107 | 0.0104 | 0.0102 | 0.0009 | 0.0093 | 0.0093 | 0.0091 | 0.0089 | 0.0087 | 0.0084 |
|  | -2.2 | 0.0139 | 0.0136 | 0.0132 | 0.0129 | 0.0125 | 0.0122 | 0.0119 | 0.0116 | 0.0113 | 0.0110 |
|  | -2, 1 | 0.0179 | 0.0174 | 0.0170 | 0.0166 | 0.0162 | 0.0158 | 0.0154 | 0.0150 | 0.0146 | 0.0143 |
|  | -2.0 -1.9 | 0.0228 | 0.0222 | 0.0217 <br> 0.0274 | 0.0212 | . 0.0207 | ${ }^{0.0222}$ | 0.0197 0.0250 0 | 0.0192 0.0244 0 | 0.0188 | 0.0183 0.0233 |
|  | -1.8 | 0.0359 | 0.0351 | 0.0344 | 0.0336 | 0.0329 | 0.0322 | 0.0314 | 0.0307 | 0.0301 | 0.0294 |
|  | 0.1 .7 | 0.0446 | 0.0436 | 0.0427 | 0.0418 | 0.0409 | 0.0401 | 0.0392 | 0.0304 | 0.0375 | 0.0367 |
|  | 4.6 | 0.0548 | 0.0537 | 0.0526 | 0.0516 | 0.0505 | 0.0495 | 0.0485 | 0.0475 | 0.0465 | 0.0455 |
|  | -1.5 | 0.0668 | 0.0655 | 0.0643 | 0.0630 | 0.0618 | 0.0606 | 0.0594 | 0.0582 | 0.0571 | 0.0559 |
|  | $\frac{.1 .4}{\text { a }}$ | 0.0008 | 0.0793 | 0.0778 | -0.0724 | 0.0749 | 0.0735 | 0.0221 | 0.0778 | 0.0594 | 0.0681 |
|  | -1.3 | 0.0968 | 0.0051 | 0.0934 0.1112 | -0.0918 | - 0.0301 | 0.0885 <br> 0.1056 <br> 0 | O. 0.089 <br> 0.1038 <br> 0 | -0.0853 | -0.0838 | 0.0823 |
|  | -1.1 | 0.1357 | 0.1335 | 0.1314 | 0.1202 | 0.1271 | 0.1251 | 0.1230 | 0.1210 | 0.1190 | 0.1170 |
|  | -1.0 | 0.1587 | 0.1562 | 0.1539 | 0.1515 | 0.1492 | 0.1469 | 0.1446 | 0.1423 | 0.1401 | 0.1379 |
|  | -0.9 | 0.1841 | 0.1814 | 0.1788 | 0.1762 | 0.1736 | 0.1711 | 0.1685 | 0.1660 | 0.1635 | 0.1611 |
|  | -0.8 | 0.2119 | 0.2090 | 0.2061 | 0.2033 | 0.2005 | 0.1977 | 0.1949 | 0.1922 | 0.1894 | 0.1887 |
|  | -0.7 | 0.2420 | 0.2389 0.2709 | -0.2358 | 0.2327 <br> 0.2943 <br> 0.0 | -0.2296 | ${ }^{0.2266}$ | 0.2336 | $\bigcirc$ | 02177 | 0.2148 |
|  | -0.6 | 0.2743 0.3085 | 0.2709 0.3050 | 0.2676 0.3015 | 0.2643 <br> 0.2981 | 0.2611 0.2946 | 0.2578 0.2912 | 0.2546 0.2877 | 0.2514 0.2843 | 02483 02810 | 0.2451 0.2776 |
|  | -0.4 | 0.3446 | 0.3469 | 0.3372 | 0.3336 | 0.3300 | 0.3284 | 0.3228 | 0.3192 | 0.3156 | 0.3121 |
|  | -0.3 | 0.3821 | 0.3783 | 0.3745 | 0.3707 | 0.3669 | 0.3632 | 0.3594 | 0.3557 | 0.3520 |  |
|  | -0.2 | 0.4207 | 0.4168 | 0.4129 | 0.4090 | 0.4052 | 0.4013 | 0.3974 | 0.3936 | 0.3897 | 0.3859 |
|  | 0.1 | 0.4602 | 0.4562 0.4960 | 4522 | 0.4483 0.4880 | 0.4443 0.4840 | 0.4404 0.4801 | 0.43 | ${ }_{0}^{0.4325}$ | 0.4 | 0.4247 0.4641 |
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