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Elementary Statistics

A Step by Step Approach **Seventh Edition**

by Allan G. Bluman http://www.mhhe.com/math/stat/blumanbrief

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CHAPTER 6 Notes The Normal Distribution ed Alghamdi, Statistics Department, Faculty of Sciences, King Abdulaziz Un

Objectives

□ Identifying distributions as symmetrical or skewed.

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- □ Identifying the properties of the normal distribution.
- □ Finding the area under the standard normal distribution, given various z values.
- □ Finding the probability of a normally distributed variable by transforming it into a standard normal variable.
- □ Finding specific data values for given percentages using the standard normal distribution.
- □ Using the central limit theorem to solve problems involving sample means for large samples. Dr. Saeed Alghamdi, Statistics Department, Faculty of Sciences, King Abd

Notes

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Normal and Skewed Distributions

- □ A *normal distribution* is a continuous, symmetric, bell-shaped distribution of a variable.
- □ If the data values are evenly distributed about the mean, the distribution is said to be *symmetrical*. (mean = median = mode)
- □ If the majority of the data values fall to the left or right of the mean, the distribution is said to be *skewed*.

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□ See Figures 6-1 and 6-2 page 301. Dr. Saeed Alghamdi, Statistics Department, Faculty of Science

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Normal Distribution Properties

- The mean, median, and mode of the normal distribution are equal and located at the center of the distribution.
- □ The normal distribution curve is <u>unimodal</u> (i.e., it has only one mode).
- \square The curve of the normal distribution is continuous, i.e., there are no gaps. Thus, for each value of *X*, there is a corresponding value of *Y*.

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Normal Distribution Properties

- □ The total area under the normal distribution curve is equal to 1.00 or 100%.
- □ The area under the normal curve that lies within ✓ one standard deviation of the mean is
 - approximately 0.68 (68%).
 - v two standard deviations of the mean is approximately 0.95 (95%).
 - three standard deviations of the mean is approximately 0.997 (99.7%).

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Area Under the Standard Normal distribution Curve

 $\hfill\square$ Note the following

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 $P(z <\!\! a) \!=\! P(z \leq\!\! a)$

 $P(a < z < b) = P(a \le z \le b) = P(z < b) - P(z < a)$

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 $P(z > a) = P(z \ge a) = 1 - P(z < a)$



- If the income of 10000 family follows a normal distribution with mean 1800 SAR and standard deviation 300 SAR, find
- The probability of a family income is less than 2550.

$$P(X < 2550) = P\left(\frac{X - \mu}{\sigma} < \frac{2550 - 1800}{300}\right) = P(z < 2.5) = 0.9938$$

• The probability of a family income is less than 1300.

$$P(X < 1200) = P\left(\frac{X - \mu}{\sigma} < \frac{1300 - 1800}{300}\right) = P(z < -1.67) = 0.0475$$

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• The probability of a family income is greater than 2400.

$$P(X > 2400) = P\left(\frac{X - \mu}{\sigma} > \frac{2400 - 1800}{300}\right) = P(z > 2)$$
$$= 1 - P(z < 2) = 1 - 0.9772 = 0.0228$$



· If the income of 10000 family follows a normal distribution with mean 1800 SAR and standard deviation 300 SAR, find • The probability of a family income is greater than 1500. $P(X > 1500) = P\left(\frac{X - \mu}{\sigma} > \frac{1500 - 1800}{300}\right) = P(z > -1)$

=1-P(z < -1) = 1-0.1587 = 0.8413• The probability of a family income is between 1650 and 2250, $P(1650 < X < 2250) = P\left(\frac{1650 - 1800}{200} < \frac{X - \mu}{2} < \frac{2250 - 1800}{200}\right)$ 300

= 0.9332 - 0.3085 = 0.6247• The number of families that have income greater than 1500, $P(X > 1500) \times 1000 = 0.8413 \times 10000 = 8413$ family Dr. Saeed Alghamdi, Statistics Department, Faculty of Sciences, King Abdulaziz Universit

σ

= P(-0.5 < z < 1.5) = P(z < 1.5) - P(z < -0.5)

300



- The lifetime of a one type of microwaves follows a normal distribution with mean 3 years and standard deviation 1 year. If one microwave was chosen randomly,
- What is the probability that its lifetime will be greater than 2 years?

$$P(X > 2) = P\left(\frac{X - \mu}{\sigma} > \frac{2 - 3}{1}\right) = P(z > -1)$$

= 1 - P(z < -1) = 1 - 0.1587 = 0.8413

· If the microwaves have warranty for one year, what is the percentage of microwaves that the factory has to exchange with new ones.

$$P(X < 1) = P\left(\frac{X - \mu}{\sigma} < \frac{1 - 3}{1}\right) = P(z < -2) = 0.0228 = 2.28\%$$

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Calculating the Value of X
When one must find the value of X, the following formula can be used:

$$X = z \cdot \sigma + \mu$$

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• To qualify for a police academy, candidates must score in the top 10% on a general abilities test. The test has a mean of 200 and a standard deviation of 20. Find the lowest possible score to qualify. Assume the test scores are normally distributed. • The test value X that cuts off the upper 10% of the area under a normal distribution curve is desired.

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0.9

0.1

X







- To qualify for a police academy, candidates must score in the top 10% on a general abilities test. The test has a mean of 200 and a standard deviation of 20. Find the lowest possible score to qualify. Assume the test scores are normally distributed.
- Now, substitute in the formula $X = z \cdot \sigma + \mu$, thus

$X = 1.28 \times 20 + 200 = 225.60 \approx 226$

• Hence, a score of 226 should be used as a cutoff. Anybody scoring 226 or higher qualifies.

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- For a medical study, a researcher wishes to select people in the middle 60% of the population based on blood pressure. If the mean systolic blood pressure is 120 and the standard deviation is 8, find the upper and lower readings that would qualify people to participate in the study.
- Note that two values are needed, one above the mean and one below the mean.







 Now, the closest *z* value for an area of 0.8000 (0.7995) is 0.84. Thus, X₁ = -0.84×8+120=113.28

$X_2 = 0.84 \times 8 + 120 = 126.72$

• Therefore, the middle 60% will have blood pressure reading between 113.28 and 126.72.

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Distribution of Sample Means

- □ A *sampling distribution of sample means* is a distribution obtained by using the means computed from random samples of a specific size taken from a population.
- □ <u>Sampling error</u> is the difference between the sample measure and the corresponding population measure due to the fact that the sample is not a perfect representation of the population.

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The Central Limit Theorem

- □ As the sample size *n* increases, the shape of the distribution of the sample means taken with replacement from a population with mean μ and standard deviation σ will approach a normal distribution.
- □ Thus, the mean of the sample means equals the population mean, $\mu_{\pi} = \mu$, and the standard deviation of the sample means which is called the *standard error of* the mean is $\sigma_{\pi} = \frac{\sigma}{\sqrt{n}}$

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The Central Limit Theorem

- The central limit theorem can be used to answer questions about sample means in the same manner that the normal distribution can be used to answer questions about individual values.
- \square A new formula must be used for the *z* values:

$$z = \frac{\overline{x} - \mu}{\sigma / \sqrt{n}}$$

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Notes

6-23 · A.C. Neilsen reported that children between the ages of 2 and 5 watch an average of 25 hours of TV per week. Assume the variable is normally distributed and the standard deviation is 3 hours. If 32 children between the ages of 2 and 5 are randomly selected, find the probability that the mean of the number of hours they watch TV is greater than 26.3 hours. $P(\bar{x} > 26.3) = P\left(\frac{\bar{x} - \mu}{\sigma/\sqrt{n}} > \frac{26.3 - 25}{3/\sqrt{32}}\right) = P(z > 2.45)$

(z < 2.45) = 1 - 0.9929 = 0.0071



The average age of a vehicle registered in the United States is 96 months. Assume the standard deviation is 16 months. If a random sample of 36 cars is selected, find the probability that the mean of their age is between 90 and 100 months.
The *z* values are

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 $P(90 < \bar{x} < 100) = P\left(\frac{90 - 96}{16/\sqrt{36}} < \frac{\bar{x} - \mu}{\sigma/\sqrt{n}} < \frac{100 - 96}{16/\sqrt{36}}\right)$ = P(-2.25 < z < 1.5) = P(z < 1.5) - P(z < -2.25)= 0.9332 - 0.0122 = 0.921



• The average number of pounds of meat that a person consumes a year is 218.4 pounds. Assume that the standard deviation is 25 pounds and the distribution is approximately normal.

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• Find the probability that a person selected at random consumes less than 224 pounds per year.

$$P(x < 224) = P\left(\frac{x - \mu}{\sigma} < \frac{224 - 218.4}{25}\right) = P(z < 0.22) = 0.5871$$

• If a sample of 40 individual is selected, find the probability that the mean of the sample will be less than 224 pounds per year.

$$P(\overline{x} < 224) = P\left(\frac{\overline{x} - \mu}{\sigma/\sqrt{n}} < \frac{224 - 218.4}{25/\sqrt{40}}\right) = P(z < 1.42) = 0.9222$$

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Notes

Summary

- □ The normal distribution can be used to describe a variety of variables, such as heights, weights, and temperatures.
- □ The normal distribution is bell-shaped, unimodal, symmetric, and continuous; its mean, median, and mode are equal.
- □ The normal distribution can be used to approximate other distributions.
- □ Mathematicians use the standard normal distribution which has a mean of 0 and a standard deviation of 1.

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Summary

The normal distribution can be used to describe a sampling distribution of sample means. These samples must be of the same size and randomly selected with replacement from the population.

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- □ The central limit theorem states that as the size of the samples increases, the distribution of sample means will be approximately normal.
- □ The distribution of sample means is much less variable than the distribution of individual data value.

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6.28		0.00	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
0-20	-3.5	0.0002	0.0002	0.0002	0.0002	0.0002	0.0002	0.0002	0.0002	0.0002	0.0002
_	-3.4	0.0003	0.0003	0.0003	0.0003	0.0003	0.0003	0.0003	0.0003	0.0003	0.0002
<u>e</u> .	-3.3	0.0005	0.0005	0.0005	0.0004	0.0004	0.0004	0.0004	0.0004	0.0004	0.0003
14	-3.2	0.0007	0.0007	0.0006	0.0006	0.0006	0.0006	0.0006	0.0005	0.0005	0.0005
ق	-3.1	0.0010	0.0009	0.0009	0.0009	0.0008	0.0008	0.0008	0.0008	0.0007	0.0007
Ξ	-3.0	0.0013	0.0013	0.0013	0.0012	0.0012	0.0011	0.0011	0.0011	0.0010	0.0010
st	-2.9	0.0019	0.0018	0.0018	0.0017	0.0016	0.0016	0.0015	0.0015	0.0014	0.0014
ö	-2.8	0.0026	0.0025	0.0024	0.0023	0.0023	0.0022	0.0021	0.0021	0.0020	0.0019
=	-2.7	0.0035	0.0034	0.0033	0.0032	0.0031	0.0030	0.0029	0.0028	0.0027	0.0026
9	-2.6	0.0047	0.0045	0.0044	0.0043	0.0041	0.0040	0.0039	0.0038	0.0037	0.0036
1 8 1	-2.5	0.0062	0.0060	0.0059	0.0057	0.0055	0.0054	0.0052	0.0051	0.0049	0.0048
ō	-2.4	0.0082	0.0080	0.0078	0.0075	0.0073	0.0071	0.0069	0.0068	0.0066	0.0064
z	-2.3	0.0107	0.0104	0.0102	0.0099	0.0096	0.0094	0.0091	0.0089	0.0087	0.0084
σ	-2.2	0.0139	0.0136	0.0132	0.0129	0.0125	0.0122	0.0119	0.0116	0.0113	0.0110
a l	-2.1	0.0179	0.0174	0.0170	0.0166	0.0162	0.0158	0.0154	0.0150	0.0146	0.0143
σ	-2.0	0.0228	0.0222	0.0217	0.0212	0.0207	0.0202	0.0197	0.0192	0.0188	0.0183
2	-1.9	0.0287	0.0281	0.0274	0.0268	0.0262	0.0256	0.0250	0.0244	0.0239	0.0233
ů,	-1.8	0.0359	0.0351	0.0344	0.0336	0.0329	0.0322	0.0314	0.0307	0.0301	0.0294
ŝ	-1.7	0.0446	0.0436	0.0427	0.0418	0.0409	0.0401	0.0392	0.0384	0.0375	0.0367
s –	-1.6	0.0548	0.0537	0.0526	0.0516	0.0505	0.0495	0.0485	0.0475	0.0465	0.0455
÷	-1.5	0.0668	0.0655	0.0643	0.0630	0.0618	0.0606	0.0594	0.0582	0.0571	0.0559
<u>a</u>	-1.4	0.0808	0.0793	0.0778	0.0764	0.0749	0.0735	0.0721	0.0708	0.0694	0.0681
2	-1.3	0.0968	0.0951	0.0934	0.0918	0.0901	0.0885	0.0869	0.0853	0.0838	0.0823
5	-1.2	0.1151	0.1131	0.1112	0.1093	0.1075	0.1056	0.1038	0.1020	0.1003	0.0985
3	-1.1	0.1357	0.1335	0.1314	0.1292	0.1271	0.1251	0.1230	0.1210	0.1190	0.1170
	-1.0	0.1587	0.1562	0.1539	0.1515	0.1492	0.1469	0.1446	0.1423	0.1401	0.1379
ĕ	-0.9	0.1841	0.1814	0.1788	0.1762	0.1736	0.1711	0.1685	0.1660	0.1635	0.1611
Ŧ	-0.8	0.2119	0.2090	0.2061	0.2033	0.2005	0.1977	0.1949	0.1922	0.1894	0.1867
5	-0.7	0.2420	0.2389	0.2358	0.2327	0.2296	0.2266	0.2236	0.2206	0.2177	0.2148
	-0.6	0.2743	0.2709	0.2676	0.2643	0.2611	0.2578	0.2546	0.2514	0.2483	0.2451
- ×	-0.5	0.3085	0.3050	0.3015	0.2981	0.2946	0.2912	0.2877	0.2843	0.2810	0.2776
a l	-0.4	0.3446	0.3409	0.3372	0.3336	0.3300	0.3264	0.3228	0.3192	0.3156	0.3121
μË	-0.3	0.3821	0.3783	0.3745	0.3707	0.3669	0.3632	0.3594	0.3557	0.3520	0.3483
e	-0.2	0.4207	0.4168	0.4129	0.4090	0.4052	0.4013	0.3974	0.3936	0.3897	0.3859
1 8	-0.1	0.4602	0.4562	0,4522	0,4483	0.4443	0.4404	0.4364	0.4325	0.4286	0.4247
	-0.0	0.5000	0.4960	0.4920	0.4880	0.4840	0.4801	0.4761	0.4721	0.4681	0.4641

6.29		0.00	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
- 27	0.0	0.5000	0.5040	0.5080	0.5120	0.5160	0.5199	0.5239	0.5279	0.5319	0.5359
_	0.1	0.5398	0.5438	0.5478	0.5517	0.5557	0.5596	0.5636	0.5675	0.5714	0.5753
	0.2	0.5793	0.5832	0.5871	0.5910	0.5948	0.5987	0.6026	0.6064	0.6103	0.6141
벅	0.3	0.6179	0.6217	0.6255	0.6293	0.6331	0.6368	0.6406	0.6443	0.6480	0.6517
ā	0.4	0.6554	0.6591	0.6628	0.6664	0.6700	0.6736	0.6772	0.6808	0.6844	0.6879
Ξ I	0.5	0.6915	0.6950	0.6985	0.7019	0.7054	0.7088	0.7123	0.7157	0.7190	0.7224
ts	0.6	0.7257	0.7291	0.7324	0.7357	0.7389	0.7422	0.7454	0,7486	0.7517	0.7549
ΞI	0.7	0.7580	0.7611	0.7642	0.7673	0.7704	0.7734	0.7764	0.7794	0.7823	0.7852
Ξ.	0.8	0.7881	0.7910	0.7939	0.7967	0.7995	0.8023	0.8051	0.8078	0.8106	0.8133
E	0.9	0.8159	0.8186	0.8212	0.8238	0.8264	0.8289	0.8315	0.8340	0.8365	0.8389
5	1.0	0.8413	0.8438	0.8461	0.8485	0.8508	0.8531	0.8554	0.8577	0.8599	0.8621
ōΙ	1.1	0.8643	0.8665	0.8686	0.8708	0.8729	0.8749	0.8770	0.8790	0.8810	0.8830
z	1.2	0.8849	0.8869	0.8888	0.8907	0.8925	0.8944	0.8962	0.8980	0.8997	0.9015
- 5	1.3	0.9032	0.9049	0.9066	0.9082	0.9099	0.9115	0.9131	0.9147	0.9162	0.9177
E	1.4	0.9192	0.9207	0.9222	0.9236	0.9251	0.9265	0.9279	0.9292	0.9306	0.9319
ΰI	1.5	0.9332	0.9345	0.9357	0.9370	0.9382	0.9394	0.9406	0.9418	0.9429	0.9441
<u> </u>	1.6	0.9452	0.9463	0.9474	0.9484	0.9495	0.9505	0.9515	0.9525	0.9535	0.9545
ا ت	1.7	0.9554	0.9564	0.9573	0.9582	0.9591	0.9599	0.9608	0.9616	0.9625	0.9633
s	1.8	0.9641	0.9649	0.9656	0.9664	0.9671	0.9678	0.9686	0.9693	0.9699	0.9706
έ	1.9	0.9713	0.9719	0.9726	0.9732	0.9738	0.9744	0.9750	0.9756	0.9761	0.9767
:≦	2.0	0.9772	0.9778	0.9783	0.9788	0.9793	0.9798	0.9803	0.9808	0.9812	0.9817
đ	2.1	0.9821	0.9826	0.9830	0.9834	0.9838	0.9842	0.9846	0.9850	0.9854	0.9857
3	2.2	0.9861	0.9864	0.9868	0.9871	0.9875	0.9878	0.9881	0.9884	0.9887	0.9890
ε	2.3	0.9893	0.9896	0.9898	0.9901	0.9904	0.9906	0.9909	0.9911	0.9913	0.9916
3	2.4	0.9918	0.9920	0.9922	0.9925	0.9927	0.9929	0.9931	0.9932	0.9934	0.9936
	2.5	0.9938	0.9940	0.9941	0.9943	0.9945	0.9946	0.9948	0.9949	0.9951	0.9952
ē	2.6	0.9953	0.9955	0.9956	0.9957	0.9959	0.9960	0.9961	0.9962	0.9963	0.9964
÷	2.7	0.9965	0.9966	0.9967	0.9968	0.9969	0.9970	0.9971	0.9972	0.9973	0.9974
5	2.8	0.9974	0.9975	0.9976	0.9977	0.9977	0.9978	0.9979	0.9979	0.9980	0.9981
0	2.9	0.9981	0.9982	0.9982	0.9983	0.9984	0.9984	0.9985	0.9985	0.9986	0.9986
<u>e</u>	3.0	0.9987	0.9987	0.9987	0.9988	0.9988	0.9989	0.9989	0.9989	0.9990	0.9990
9	3.1	0.9990	0.9991	0.9991	0.9991	0.9992	0.9992	0.9992	0.9992	0.9993	0.9993
μË	3.2	0.9993	0.9993	0.9994	0.9994	0.9994	0.9994	0.9994	0.9995	0.9995	0.9995
e	3.3	0.9995	0.9995	0.9995	0.9996	0.9996	0.9996	0.9996	0.9996	0.9996	0.9997
_ ع	3.4	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9998
-	3.5	0.9998	0.9998	0.9998	0.9998	0.9998	0.9998	0.9998	0.9998	0.9998	0.9998